Algorithm of 3D Spatial Coordinates Measurement Using a Camera Image

Hyeon Min Lee and Woo Chun Choi
Department of Mechanical Engineering, Korea University, Seoul, Korea
Email: wcchoi@korea.ac.kr

Abstract—In many researches for image processing technology, 3D coordinates measurement can be done by using laser range finders or stereo vision sensors. However, it has a disadvantage of high cost and large and heavy sensors. This study proposes an algorithm of transformation of 2D positions in a camera image to 3D spatial coordinate values. The coordinates of 3 points have to be measured in advance, in order to obtain the coordinate values of unknown points. The proposed algorithm was verified through experiment. This algorithm can be used in modern industry measurement systems and development of weapon systems.

Index Terms—3D measurement, coordinate, monocular camera

I. INTRODUCTION

There is widespread interest in obtaining 3D data by means of machines for the purpose of studies and applications of machine perception, pattern recognition, computer vision, robotic vision, computer aided design, apparel design, and so on. Three-dimensional data would serve to solve a wide variety of problems in such fields easily, as technology of image processing develops with computer operation speed, many researches have been done on measurement using cameras instead of human eyes. Technology of 3D measurement, image processing and recognition has been developed in industry for precision manufacturing. Among various methods, optical 3D measurement techniques have been used in many fields such as reverse engineering, quality inspection, etc. [1] A typical optical coordinate measurement is local mapping using laser range finders, which has been investigated actively.

Recently, sensors such as laser range finder, camera sensor, ultrasonic sensor and Kinect are used to perform local mapping. Although a laser range finder has high accuracy and reliability, it is not appropriate to be used in real industry, because it is very expensive. [2] Also, there is a stereo vision used widely for 3D measurement. This realizes the function of recognition of 3D space through human eyes using 2 cameras. The principle of this technology is that 3D measurement is done using 2 or more images by adjusting points in the images. [3], [4]

The monocular camera algorithm method gives 3D measurement data via a mush simpler computational scheme, so it may be more promising at this state of the art. The conventional monocular camera methods, however rely on the structured light such as CMM or SLS. Hence the SLS consists of a projector together with a camera. The projector projects a coded stripe pattern on the scene and the camera captures an image. Hence for each visible point in the world there is a corresponding stripe number (stripe value) and image location (pixel coordinates). Given the parameters of the SLS each stripe value defines a plane in the world and each pixel defines a ray in the world. The intersection of this ray and plane defines a unique 3D location in the world. The parameters of the SLS are obtained during system calibration. This is achieved by presenting a calibration reference in the field of view which features a set of fiducial marks whose spatial location is known to high accuracy. The SLS is operated normally and the pixel coordinates and stripe value for each of the fiducial marks are obtained. These triples of reference, pixel and stripe coordinates are used to estimate the unknown parameters of a mathematical model of the SLS. In this way calibration can be regarded as a parameter estimation problem. [5]-[10]

As mentioned previously, existing technologies of 3D measurement require expensive devices and complicated processes. However, transformation of 2D images or videos to 3D coordinates does not need those requirements. It can be used in 3D measurement of MAV (Micro Air Vehicle), and object trajectory measurement in weapon tests.

In this study, an algorithm is proposed that 3D coordinates are obtained from a 2D camera image.
II. HELPFUL HINTS

To assume that the coordinates of the origin (current camera position $p_0$ in Fig. 1) and 3 points ($p_2$, $p_3$, $p_4$) are known. The coordinates of a point are obtained from the relation between the coordinates of 2D images and 3D points. The unit vectors from $p_0$ to $p_2$ and from $p_0$ to $p_3$ in Fig. 1 are

$$e_2 = \frac{p_2 - p_0}{|p_2 - p_0|}, \quad e_3 = \frac{p_3 - p_0}{|p_3 - p_0|} \quad (1)$$

The unit vectors, $e_2, e_3$, are deviated from a vertical axis by $\alpha_2 = \tan^{-1}(r_2 / a_0), \alpha_3 = \tan^{-1}(r_3 / a_0)$ and as shown in Fig. 2, respectively. $a_0$ is the pixel focal length. From the two unit vectors, a unit vector corresponding to $e_2$ from $p_0$ and $e_3$, can be determined.

$$e_{23} = \frac{\sin \alpha_2 e_2 - \sin \alpha_3 e_3}{\sin \alpha_2 - \sin \alpha_3} \quad (2)$$

Similarly, unit vectors can be obtained.

$$e_{24} = \frac{\sin \alpha_2 e_2 - \sin \alpha_4 e_4}{\sin \alpha_2 - \sin \alpha_4} \quad (3)$$

$$e_{23}, e_{24}$$ are deviated from a horizontal axis by $\beta_2, \beta_3$ respectively. $\beta_2, \beta_3$ can be obtained as

$$s_{23} = \frac{\sin \alpha s_2 - \sin \alpha s_3}{\sin \alpha_2 - \sin \alpha_3} \quad (4)$$

$$s_{24} = \frac{\sin \alpha s_2 - \sin \alpha s_4}{\sin \alpha_2 - \sin \alpha_4}$$

$$\beta_2 = \tan^{-1}(s_{23} / a_0)$$

$$\beta_3 = \tan^{-1}(s_{24} / a_0)$$

From $\beta_2, \beta_3$, a unit vector connecting $p_0$ and $q_0$ can be obtained. This unit vector is normal to the camera picture plane.

An equation of a plane is expressed as $ax + by + cz = 0$, and coefficients $a, b, c, d$ can be represented by a normal vector and a point in the plane

$$e_{01}x + e_{02}y + e_{03}z - (e_{01}q_{0x} + e_{02}q_{0y} + e_{03}q_{0z}) = 0 \quad (6)$$

$$q_0 = p_0 + ma_0 e_0 = (p_{0x}, p_{0y}, p_{0z}) + ma_0 (e_{0x}, e_{0y}, e_{0z}) \quad (7)$$

Point $q_0$ is represented by equation (7). $m$ is a magnification factor which transforms a pixel value to a spatial length. The line connecting $p_0$ and $q_i$ meets the plane at point $q_i$ as shown in Fig. 2. $q_2$ is located in the direction of $e_2$ from $p_0$, and can be expressed as using a parameter $t$.

$$q_2 = p_0 + te_2 = (p_{0x}, p_{0y}, p_{0z}) + t(e_{2x}, e_{2y}, e_{2z}) \quad (8)$$

This point is on the plane, $ax + by + cz = 0$

$$e_{01}(p_{0x} + te_{2x}) + e_{02}(p_{0y} + te_{2y}) + e_{03}(p_{0z} + te_{2z}) = 0$$

$$-e_{01}(p_{0x} + ma_0 e_{0x}) + e_{02}(p_{0y} + ma_0 e_{0y}) + e_{03}(p_{0z} + ma_0 e_{0z}) = 0 \quad (9)$$

From equation (9), the parameter $t$ can be determined as

$$t = \frac{ma_0 (e_{01}^2 + e_{02}^2 + e_{03}^2)}{e_{01} e_2 + e_{02} e_2 + e_{03} e_2} \quad (10)$$

$q_i$ can be determined by substituting $t$ into equation (8).

There exists a transformation relation between the picture plane and the real plane. A transformation array $J$ is

$$rs = Jq \quad (11)$$

For instance, the following equation holds for $i = 2, 3, 4$.

$$[r_{i1}, r_{i2}, r_{i3}, r_{i4}, r_{i5}, r_{i6}, r_{i7}, r_{i8}, r_{i9}] \begin{bmatrix} J_{11} & J_{12} & J_{13} & q_{0x} \\ J_{21} & J_{22} & J_{23} & q_{0y} \\ J_{31} & J_{32} & J_{33} & q_{0z} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_{i1} \\ r_{i2} \\ r_{i3} \\ r_{i4} \\ r_{i5} \\ r_{i6} \\ r_{i7} \\ r_{i8} \\ r_{i9} \end{bmatrix}$$

Or

$$r_2 = J_{11}q_{2x} + J_{21}q_{3x} + J_{31}q_{4x}$$

$$r_3 = J_{12}q_{2y} + J_{22}q_{3y} + J_{32}q_{4y}$$

$$r_4 = J_{13}q_{2z} + J_{23}q_{3z} + J_{33}q_{4z}$$

$$s_3 = J_{11}q_{3x} + J_{21}q_{3y} + J_{31}q_{3z}$$

$$s_4 = J_{12}q_{3y} + J_{22}q_{3z} + J_{32}q_{3z}$$

$$c = J_{13}q_{3z} + J_{23}q_{3z} + J_{33}q_{3z}$$

$$s_1 = J_{11}q_{1x} + J_{21}q_{1y} + J_{31}q_{1z}$$

$$s_2 = J_{12}q_{1y} + J_{22}q_{1z} + J_{32}q_{1z}$$

$$s_3 = J_{13}q_{1z} + J_{23}q_{1z} + J_{33}q_{1z}$$

$$c = J_{13}q_{1z} + J_{23}q_{1z} + J_{33}q_{1z}$$

$$e_0 = \frac{\sin \beta_1 e_{0x} - \sin \beta_1 e_{23}}{\sin \beta_1 - \sin \beta_3} \quad (5)$$
$\begin{bmatrix} q_2, q_2, q_2; \\ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ q_3, q_3, q_3; \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ \vdots \ \vdots \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ J_{11} \\ J_{21} \\ J_{31} \\ J_{12} \\ J_{22} \\ J_{32} \\ J_{13} \\ J_{23} \\ J_{33} \end{bmatrix} = \begin{bmatrix} J_{11} \\ J_{21} \\ J_{31} \\ J_{12} \\ J_{22} \\ J_{32} \\ J_{13} \\ J_{23} \\ J_{33} \end{bmatrix} = \begin{bmatrix} q_2 \\ q_2 \\ q_2 \\ q_3 \\ q_3 \\ q_3 \\ q_4 \\ q_4 \\ q_4 \end{bmatrix}$

(14)

$r, s, q$ values are obtained from the pixel values. From equation (14), $J$ can be determined. $q$ can be determined by substituting $r, s$ into equation (11). For example, putting the pixel coordinates $r_5, s_5$ into equation (3) corresponding to the point $q_5$ in Fig. 3, $q_5$ can be determined.

$p_5$ can be obtained by determining a unit vector $e_5$ connecting $p_0$ and $q_5$

$p_5 = p_0 + t e_5 = (p_{0x}, p_{0y}, p_{0z}) + t(e_{5x}, e_{5y}, e_{5z})$  \hspace{1cm} (16)

When $p_5$ is located on plane $z=0$, then $t$ is obtained as

$p_{5z} = p_{0z} + t e_{5z} = 0$

$t = -\frac{p_{0z}}{e_{5z}}$  \hspace{1cm} (17)

### III. EXPERIMENT AND RESULT

From 2D pixel coordinates, 3D coordinates are to be obtained using a MATLAB code based on linear algebra. The experiment was done as shown in Fig. 3.

#### TABLE I. REAL COORDINATE VALUES (UNIT: MM)

<table>
<thead>
<tr>
<th>Point</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_2$</td>
<td>-431.3</td>
<td>3194.6</td>
<td>1240</td>
</tr>
<tr>
<td>$p_3$</td>
<td>30</td>
<td>3626</td>
<td>630</td>
</tr>
<tr>
<td>$p_4$</td>
<td>0</td>
<td>3626</td>
<td>1240</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-315</td>
<td>3180</td>
<td>820</td>
</tr>
</tbody>
</table>

#### TABLE II. PIXEL COORDINATE VALUES (UNIT: MM)

<table>
<thead>
<tr>
<th>Point</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_2$</td>
<td>320</td>
<td>486</td>
<td>0</td>
</tr>
<tr>
<td>$p_3$</td>
<td>21</td>
<td>64</td>
<td>0</td>
</tr>
<tr>
<td>$p_4$</td>
<td>4</td>
<td>475</td>
<td>0</td>
</tr>
<tr>
<td>$p_5$</td>
<td>-236</td>
<td>163</td>
<td>0</td>
</tr>
</tbody>
</table>

For verification of the proposed algorithm, experiment was done with 4 points selected as shown in Fig. 4. The four points were on the same plane. The coordinates of the 4 points were measured in 3D real space. The results are shown in Table I. The camera position was measured from a plane $z=0$. A camera picture was taken with all the points in a frame. From the camera image, the pixel coordinates were measured as shown in Table II.

The pixel values and coordinated of all the points were input to the equations, and the 3D coordinates of was obtained. The distance of the points from the camera was 4m. The results are shown in Table III. The error of the measured coordinates is about 1.7%, compared with the real coordinates. This shows that the proposed algorithm is good enough.

#### TABLE III. RESULT OF EXPERIMENT (UNIT: MM)

<table>
<thead>
<tr>
<th>Point</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real  $p_5$</td>
<td>-315</td>
<td>3180</td>
<td>820</td>
<td>1.7%</td>
</tr>
<tr>
<td>Computed $p_5$</td>
<td>-314</td>
<td>3085</td>
<td>837.5</td>
<td></td>
</tr>
</tbody>
</table>

#### IV. CONCLUSIONS

In this study, an algorithm of determining 3D coordinates from 2D image was proposed. Experimental results showed that the measurement has about 1.7% error. This means that the algorithm is good enough. Using this algorithm, economical and fast measurement is possible.
REFERENCES


Hyun Min Lee completed his B.S from Seoul National University of Science and Technology, Korea in Department of Mechanical engineering, in 2013. He is currently pursuing the MS degree in the Precision Technology Lab. of the Graduate School of Mechanical Engineering at Korea University, Korea. His study includes measurement of 3D spatial coordinates by 2D image, image processing.