Noise Removal Method for Moving Images Using 3-D and Time-Domain Total Variation Regularization Decomposition

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Abstract—In recent years, in order to display high vision broadcast the next generation displays, super resolution techniques for improving image resolution are demanded. In addition, with the spread of digital cameras and smartphones, people have more opportunities to handle camera images. In particular, images of surveillance cameras are required to obtain high-definition output by removing noise. In this paper, in order to avoid the adverse effect of image quality deterioration when emphasizing noise mixed image which is a problem of super resolution processing, we examine a noise removal method before super resolution processing. In our proposed method, Total Variation regularization, which is decomposed into structure and texture components, is extended in direction of time axis. As a result, moving images can be decomposed into structure moving images and texture moving images. In theory, it is thought that noise components with large value of Total Variation should shift to texture components. Furthermore, we aim for separation of texture components and noise, and aim for acquisition of high-definition moving images. We verify the performance of our proposed method by comparing it with the BM3D method, which is regarded as the highest performance for moving image noise removal processing.

Index Terms—total variation regularization, noise removal method, bm3d method

I. INTRODUCTION

In recent years, resolutions of display devices have been increased. In general, Full HD (1920×1080 pixels) is widely used as the resolution of display devices but displays with 4K (3840×2160 pixels) have been also spreading to general households as a movement to further increase the resolution. In addition, research and development of 8K (7680×4320 pixel) displays called Super Hi-Vision, which is further advanced, is progressing and it is expected that displays will be spread toward the 2020 Tokyo Olympic Games.

On the other hand, consumer video cameras are now readily available with high performance and low price, so people are using them to create various contents. Also, smartphones are equipped with high-performance video cameras, people are free to shoot videos and enjoy themselves.

Among these moving picture contents, performance improvement of surveillance cameras is considered to be the most important. Several super resolution technologies have been proposed to make images of surveillance camera high definition, but if noise is mixed in original images, it also emphasizes even the mixed noise. Even if the performance of image sensors is high, there is a problem that the light amount is insufficient in nighttime photographing, and noise appears in the image. From the above, it can be seen that in the case of super-resolution processing on videos of surveillance cameras, it is necessary to remove noise in advance before super-resolution processing.

In this paper, Total Variation regularization processing is used as a noise removal method before super resolution processing. This processing is a technique that can separate components of an image for each signal size. In the conventional Total Variation Regularization of the 2D image, noise separation was not successful. Therefore, in our proposed method, focusing on the magnitude of the Total Variation in time axis direction, noise separation is performed. First, in addition to spatial axis direction of Total Variation regularization of the conventional method, 3-D Total Variation regularization that uses information in time axis direction is used. Second, we use Time-Domain Total Variation regularization focusing on Total Variation only in time axis direction. Using these new Total Variation regularization, noise separation and original image retention are performed. We believe that it will be possible to improve noise removal performance more than ever by becoming able to change the noise removal method and parameters for each component.

II. TOTAL VARIATION REGULARIZATION

A. Total Variation

A structure component and a texture component in images can be decomposed. Consider using Total Variation regularization method to perform this separation [1]-[3].

Total Variation is sum of absolute values of variations of all neighboring luminance values within a certain region in an image. The Total Variation of two-dimensional signal \( u \) is represented by (1). Component decomposition of images can be achieved by paying attention to Total Variation of images.
\[ TV(u) = \int |\nabla u| \, dx \, dy \] (1)

Total Variation of components constituting an image is shown in Fig. 1. In Fig. 1, (a) is a low-frequency component whose change in luminance value is smooth. Although the absolute value change amount of the adjacent luminance value is small, it finally becomes a constant amount of total variance by accumulating it. (b) is an edge component whose luminance value sharply changes. In the edge portion, it is understood that the absolute value change amount of the luminance value becomes Total Variation as it is. (c) is a vibration component forming a fine pattern. The Total Variation becomes very large as the luminance value shakes violently. As described above, the portion constituted by vibration components shows a very large Total Variation value. (c) is a vibration component whose luminance value sharply changes. On the other hand, it can be seen that the texture component image is constituted only by the component whose TV is large and whose luminance value continuously changes.

When trying to find similar characteristics in a discrete system, we obtain (11).

\[ f(u) = \sup_{v \in K} (\langle (\nabla u), v \rangle) \] (11)

value of 128. The structure component image is an image composed of a low frequency component with poor brightness value change and an edge component with a sharp brightness value change. On the other hand, it can be seen that the texture component image is constituted only by the component whose TV is large and whose luminance value continuously changes.

As for the minimization problem of TV, A. Chambolle proposed its fast solution [4]-[6]. In this method, a dual variable is calculated by performing iterative calculation using a reduced image. In the minimization problem, the image is defined as an \( N \times N \) two-dimensional array, \( X \) is Euclidean space, \( R \) and \( Y \) are \( N \times N \) vectors. The structure image \( u \) and the texture image \( v \) use (5), for \( p = (p^1, p^2) \) and \( q = (q^1, q^2) \) \( \in Y \) use (6).

\[ (u, v)_X = \sum_{1 \leq i, j \leq N} u_{ij} v_{ij} \] (5)

\[ (p, q)_Y = \sum_{1 \leq i, j \leq N} (p_{ij}^1 q_{ij}^1 + p_{ij}^2 q_{ij}^2) \] (6)

In order to discretize the dual problem, we define TV of \( u \) as follows.

\[ f(u) = \sum_{1 \leq i, j \leq N} |(\nabla u)_{ij}| \] (7)

The Lengendre-Fenchel transformation is applied to \( f \) to get (8). Also, since \( f \) is primary homogeneous and becomes a characteristic function of the closed convex set \( K \), it becomes as shown (9).

\[ f^*(v) = \sup_{u} (\langle u, v \rangle_X - f(u)) \] (8)

\[ = \chi_K(v) = \begin{cases} 0 & (v \in K) \\ +\infty & \end{cases} \] (9)

where since \( f^* = f \), (10) is obtained when \( v \in K \).

\[ f(u) = \sup_{v \in K} (\langle (\nabla u), v \rangle) \] (10)

When trying to find similar characteristics in a discrete system, we obtain (11).

\[ f(u) = \sup_{p} \langle p, \nabla u \rangle \] (11)
The minimization problem to be found here is (12) and the Euler equation is (13).

\[
\min_{u \in \Omega} \frac{1}{2\lambda} \|u - f\|^2 + J(u) \quad (12)
\]

\[
u - f + \lambda \partial J(u) \ni 0 \quad (13)
\]

In (13), \(\partial J\) is a subderivative of \(J\), for all \(\nu \in \partial J(u) \Leftrightarrow J(u) \geq J(v) + \langle \nu, u - v \rangle \). This is a condition for \(\nu\) to be an inferior to \(J(u)\). The Euler equation of (13) is \((f - u) / \lambda \in \partial J(u)\), which is equivalent to \(u \in \partial J^*(f - u) / \lambda\). Therefore, (14) is obtained.

\[
\frac{f}{\lambda} = \frac{f - u}{\lambda} + \frac{1}{\lambda} \partial J^*(f - u) / \lambda \quad (14)
\]

We also obtain that \(\omega = (f - u) / \lambda\) is a minimal in (15).

\[
\frac{\|\omega - (f / \lambda)\|^2}{2} + \frac{1}{\lambda} \partial J^*(\omega) \quad (15)
\]

From (9), we can calculate \(\omega = \pi_{\lambda K}(f / \lambda)\). Therefore, \(u\), which is the solution of the minimization problem of (12), is obtained as shown in (16).

\[
u = f - \pi_{\lambda K}(f) \quad (16)
\]

Calculation of nonlinear projection \(\pi_{\lambda K}\) is used as an algorithm that can calculate \(u\). Calculating the nonlinear projection \(\pi_{\lambda K}\) is equivalent to solving the minimization problem of (17).

\[
\min \left\{ \|\lambda \text{div} p - f\|^2 : p \in V, |p_{i,j}| \leq 0 \; \forall i, j = 1, \ldots, N \right\} \quad (17)
\]

The Karush-Kuhn-Tucker condition yields the Lagrangian multiplier \(\alpha_{i,j} \geq 0\) related to the constraint of the minimization problem (17). Therefore, for each \(i, j\), it becomes as shown in (18), and there are two possible combinations of \(\alpha\) and \(|p_{i,j}|\) at that time as in (19).

\[
-\langle \nabla (\lambda \text{div} p - f) \rangle_{i,j} + \alpha_{i,j} |p_{i,j}| = 0 \quad (18)
\]

\[
\begin{align*}
\alpha_{i,j} &> 0, \quad |p_{i,j}| = 1 \\
\alpha_{i,j} &< 0, \quad |p_{i,j}| < 0
\end{align*} \quad (19)
\]

In the case of \(\alpha_{i,j} = 0, |p_{i,j}| < 1 \). \(\langle \nabla (\lambda \text{div} p - f) \rangle_{i,j} = 0\), which is inappropriate. Therefore, it becomes as shown in (20).

\[
\alpha_{i,j} = |\langle \nabla (\lambda \text{div} p - g) \rangle_{i,j}| \quad (20)
\]

By substituting in (20) into (18) and using the steepest descent method, (21) is obtained. However, for any \(n, \tau > 0, p^0 = 0\).

\[
\nu^{(n+1)}_{i,j} = \nu^{(n)}_{i,j} + \tau \left[ \langle |\lambda \text{div} p^{(n)} - f\rangle \rangle_{i,j} \right] - \frac{1}{\lambda} \left[ |\lambda \text{div} p^{(n)} - f\rangle \rangle_{i,j} \right] \quad (21)
\]

By transforming (21), (22) is obtained.

\[
\nu^{(n+1)}_{i,j} = \frac{\nu^{(n)}_{i,j} + \tau |\lambda \text{div} p^{(n)} - f\rangle \rangle_{i,j} + 1}{\lambda} \quad (22)
\]

where \(\tau\) is the step width of the steepest descent method. In the ROF model using the projection method of Chambolle, \(p\) is obtained by iterative calculation as shown in (23). Using this \(p, \pi_{\lambda K} = -\lambda \text{div} p\) is obtained, and a structure image and a texture image are obtained as shown in (16).

\[
\nu^{(n+1)}_{i,j} = \frac{\nu^{(n)}_{i,j} + \tau |\lambda \text{div} p^{(n)} - f\rangle \rangle_{i,j} + 1}{\lambda} \quad (23)
\]

By the projection method of Chambolle, it is possible to greatly shorten the processing time of TV regularization processing. In addition, a method for further improving the convergence speed by applying the Chambolle projection method using a model with increased information amount of TV has been proposed.

III. 3-D TOTAL VARIATION REGULARIZATION

In the previous chapter, it was confirmed that the image can be decomposed into two components by utilizing Total Variation regularization. Next, let us assume a case where photographing is performed in a place where the amount of light is small or at night, and noise is mixed in images. In the previous research, Total Variation regularization was applied to a still frame of a moving image in which noise was mixed, and the filtering process was performed after moving noise to noise components. However, in this method, it is difficult to accurately shift only noise to texture components, and noise is dispersed in both structure and texture components, and it cannot be achieved to perform effective processing for each decomposed component. Fig. 3 shows how noise appeared in each image after decomposition. Here, we use the fact that there is a strong correlation between adjacent frames for moving images. In the conventional Total Variation regularization method, since Total Variation on the frame direction was not taken into account, the time axis is added as the third axis in addition to the conventional spatial axis. It is thought that this makes it possible to separate more highly accurate structural components and texture / noise components. If noise can be completely shifted to texture components, theoretically, processing for structure components should be unnecessary.
Figure 3. Component decomposition of structure and texture

Figure 4. 3-D TV regularization decomposition

\( TV(u) \) of the signal \( u \) is shown in (1) in the case of a two-dimensional image, but it is represented by (24) in the case of a three-dimensional signal \( u \).

\[
TV(u) = \int |\nabla u| dx dy dt
\]  

(24)

Accordingly, the ROF model in the case of a three-dimensional signal can be rewritten from (2) to the following (25).

\[
F(u) = \int |\nabla u| dx dy dt + \lambda \int |f - u|^2 dx dy dt
\]  

(25)

A noisy moving image is separated by a three-dimensional Total Variation regularization method, and the state is shown in Fig. 4 as a still frame. As you can see, it is possible to extract only the structure component appropriately from noisy input, and all noise can be transferred to the texture components. This state can be read from the signal in an input image. An image of 200 × 200 size shown in Fig. 4 was scanned at a point of height 100 to examine its luminance level. The result is shown in Fig. 5.

The black line in Fig. 5 is a line with a luminance level of 128 as a reference. (a) is a noisy input moving image. It can be seen that the signal changes from a high luminance level to a low luminance level even though it contains noise. The high brightness level portion represents the bright portion of the left half in Fig. 4 and the low brightness level represents the dark portion of the right half of the image. (b) is the structure component after decomposition. The waveform of the signal gives a gentle impression, and it is possible to clearly confirm the edge portion. (c) is the texture component after decomposition. This signal is a component that vibrates up and down centering on 0, but brightness 128 is added to the whole image for visualization. Accordingly, (c) is a signal that vibrates about the luminance 128.

It turns out that this signal contains a lot of intensely moving components. As described above, it is possible to read from the image and the signal that no noise remains in the structure component. This is the meaning of "No processing is needed theoretically".

Figure 5. Component decomposition of original image

IV. TIME-DOMAIN TOTAL VARIATION REGULARIZATION

A huge amount of noise is mixed in the texture component after 3-D Total Variation regularization decomposition processing. To remove noise, methods using a Gaussian filter, or a bilateral filter have been proposed so far. However, the filtering process has a disadvantage of impairing the texture, although it has a noise elimination effect. Therefore, in this paper, we use the fact that noise moves greatly along the time direction for moving images. In the previous chapter 3-D Total Variation regularization, we have referred to the information in the spatial and temporal axis direction, but here we only refer to the information on the Total Variation on the time axis. Time domain Total Variation regularization formula is expressed as follows.
\[ TV(u) = \int |\nabla u| \, dt \quad (26) \]

\[ F(u) = \int |\nabla u| \, dt + \lambda \int |f - u|^2 \, dt \quad (27) \]

The noise-added texture is component-decomposed by the time domain Total Variation regularization method, and this state is shown in Fig. 6 as a still frame. (a) is the same as the image (c) in Fig. 4. (b) seems to be able to extract texture components cleanly. The component that does not move in the time axis direction, that is, the background portion can be reliably extracted because Total Variation becomes 0. The image is somewhat rough, but this is because relatively little motion noise has remained. For (c), most of it is noise. Although the precision of component decomposition is rougher than that in three dimensions, it seems better to use (b) rather than applying some processing directly to (a).

V. PROPOSED METHOD

A block diagram of our proposed method is shown in Fig. 7. This block diagram consists of 4 blocks: a 3-D Total Variation Regularization Block, a Time Domain Total Variation Regularization Block, a Noise Removal Block, and a Composite Block. In this figure, the Total Variation Regularization is represented by a hexagon, and noise removal is represented by a diamond. First, apply 3-D Total Variation Regularization to the noisy input video and perform component decomposition. This makes it possible to decompose an input moving image into the structure component and the texture/noise components. Next, time domain Total Variation regularization is applied to texture/noise components to extract background texture. Here we call this a "temporary texture". Regarding noise with little motion, it remains a bit in the temporary texture. In order to remove this noise, BM3D (Block-Matching and 3D filtering) [7], [8] which is a noise elimination method using a non-local average is used. Since BM3D excels in noise elimination performance to the extent that it can be compared with various noise elimination methods, in this paper we focus on its removal performance and decide to incorporate it in processing.

On the other hand, not only noise but also the movement components which are large values of the Total Variation are mixed in the noise component. Therefore, in order to extract this motion component, BM3D is also applied to the noise component. Since temporary texture has little noise, BM3D is applied weakly. On the other hand, since we need to remove enormous noise for noise components, we adopt a stronger BM3D. By performing component separation, it is possible to apply processing of strength suitable for each component. Finally, the respective components after the BM3D processing are synthesized to become a true texture component. After that, by synthesizing this true texture component and the structure component, it is possible to obtain the output image.

VI. EXPERIMENTAL RESULTS

Experiment is conducted to show the effectiveness of the proposed method. We use three kinds of moving images, LASVEGAS, CITY, and CAT as shown in Fig. 8. Each image is a color moving image composed of 50 frames of 480 × 270 pixels. Table I shows the parameters of the Total Variation regularization process used in the experiment. We obtained the result that the number of iterations of Total Variation regularization for 2D images is sufficient about 30 times from the previous study. However, in the case of moving images, it is necessary to accurately separate noise, so we decided to secure 50 iterations a little more this time. The experimental conditions are defined as follows:

(a) LASVEGAS
Figure 8. Experimental moving images

TABLE I. PARAMETERS OF EACH TOTAL VARIATION

<table>
<thead>
<tr>
<th>Processing Type</th>
<th>3-D Total Variation</th>
<th>Time-Domain Total Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iterations</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Constraint ( \lambda )</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>Step Width ( \tau )</td>
<td>0.125</td>
<td>0.125</td>
</tr>
</tbody>
</table>

I). Gaussian noise with 0.01 variance, BM3D strength 10, strength 5 for Tentative texture

II). Gaussian noise with 0.03 variance, BM3D strength 30, strength 15 for Tentative texture

III). Gaussian noise with 0.05 variance, BM3D strength 50, strength 25 for Tentative texture

As a comparison target, the processing result obtained by applying the BM3D processing directly to the noisy input is used. Parameters at this time shall be parameters 10, 30, 50 applied to noise component in our proposed method. In order to perform objective evaluation, the average PSNR between 50 frames of our proposed method and BM3D output image was calculated, and Table II shows the results of 9 patterns combined with 3 experimental conditions and 3 moving images.

TABLE II. RESULTS OF PSNR IN EACH METHOD (dB)

<table>
<thead>
<tr>
<th></th>
<th>LASVEGAS</th>
<th>CITY</th>
<th>CAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Proposed method</td>
<td>24.28</td>
<td>24.76</td>
</tr>
<tr>
<td></td>
<td>BM3D</td>
<td>23.41</td>
<td>24.22</td>
</tr>
<tr>
<td>II</td>
<td>Proposed method</td>
<td>25.61</td>
<td>24.06</td>
</tr>
<tr>
<td></td>
<td>BM3D</td>
<td>26.44</td>
<td>25.95</td>
</tr>
<tr>
<td>III</td>
<td>Proposed method</td>
<td>23.57</td>
<td>22.08</td>
</tr>
<tr>
<td></td>
<td>BM3D</td>
<td>28.42</td>
<td>27.57</td>
</tr>
</tbody>
</table>

In the case I), the PSNR for our proposed method is higher than that for BM3D method because absolute amount of noise is small, and it seems that decomposition in the Total Variation regularization process could be done accurately. In the case II), the PSNR for BM3D is higher than that for our proposed method except for the CAT moving image. Fig. 9 shows an enlarged view of a part of the still frame of the CAT movie whose performance has overcome BM3D. Although it is a small difference, noise remains in the result of BM3D. It may have been a video that happens to be weak for BM3D. In the case III), which increased noise, the PSNR was greatly deteriorated, and the proposed method became ineffective. From this, it is conceivable that the proposed method is effective while the amount of noise is small, conversely speaking, as the noise increases, some adverse effect may begin to appear.
Fig. 10 shows the PSNR in frame units under the case I) showing the superiority of our proposed method. Here, the horizontal and vertical axes are the number of frames and PSNR, respectively. As shown above, all of the moving images consist of 50 frames. In Fig. 10, it can be seen that the performance of the proposed method exceeds that of BM3D in all frames, and the PSNR fluctuates greatly in the 1st and 50th frames, this is because of the effect of boundary processing in the program.

We focus on the still images of LASVEGAS and CITY moving images of the case II) which the PSNR has degraded slightly in our proposed method. Fig. 11 and Fig. 12 show still frames enlarging a part of BM3D output and our proposed method output of LASVEGAS moving images. At the character parts in Fig. 11, an easy-to-see impression is obtained for our proposed method. In the output of BM3D, it seems that characters are broken. Also, in Fig. 12, our proposed method is clearer, although it is slightly different. Still frames of the CITY moving image are shown in Fig. 13 and Fig. 14. Particularly easy to understand is the roof of the building in Fig. 13. It can be confirmed that the pattern of the lattice that is collapsed by BM3D is clearly left in our proposed method. Also, regarding the pattern of sky in Fig. 14, BM3D receives an impression that noise is left. The common point between Fig. 11 and Fig. 14 is that it is a portion of the background does not move. It is confirmed that texture extraction can be effectively performed by Time-Domain Total Variation regularization. Even if the performance is inferior in PSNR in this way, our proposed method is more beautiful in actual images in some cases. One possible reason why the PSNR is lowered is the possibility that unnoticed noise remains in the structure component that has not been processed. In theory, processing should be unnecessary for the structure component, but if noise is too much, there is a possibility that the signal changes fundamentally from the original structure component, so it is necessary to consider. It is necessary to devise measures to improve not only the texture of the image but also the PSNR at the same time. In addition, since the parameters of BM3D and BM3D inside our proposed method are the same in this paper, better results can be obtained by setting to optimal parameters for our proposed method.

VII. CONCLUSION

In this paper, we have attempted to remove noise of moving images targeting surveillance camera images. In the proposed method, separation of noise components can be achieved by using 3-D Total Variation regularization. The state of this component separation was observed from both the image and the image signal. As a result, no noise was observed in the structure components, and it was confirmed that the structure components became smooth signal. On the other hand, it was confirmed that the signal contains a large moving component and noise component on the texture components side. In 3-D Total Variation regularization, it has been possible to extract noiseless structure components, and we have been able to achieve the goal of transferring all noise to the texture components. In Time-Domain Total Variation regularization,
regularization, we could acquire background texture based on the motion of noise in time axis direction. Although not only noise but also motion components have been mixed on noise components, the application of the BM3D method also has made it possible to hold motion components. By combining these components, it has been possible to obtain an ideal true texture component. Experimental results show that our proposed method shows superiority to BM3D performance in all images when there is little noise, but as noise increases, residual noise appears on the structure component side, and the proposed method becomes ineffective. Even when noise level is higher and the PSNR becomes lower, our proposed method sometimes shows advantages over the BM3D method. In our proposed method, there is an advantage for preserving details such as roof patterns, building textures clearly. Therefore, it is necessary to improve not only the appearance but also the PSNR, and it is required to specify the cause of PSNR deterioration. To show the effectiveness of our proposed method, it is necessary to consider improvement of the noise separation accuracy when there is much noise. Although BM3D is used as a noise removal block in this paper, it will be necessary to apply Gaussian filter, bilateral filter, and another processing. The ultimate goal is to construct a surveillance camera system that can obtain high-definition images without noise by combining the super-resolution processing, which we have done in [9]-[10].

REFERENCES


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